Elementary Maths for GMT

Answers

Exercise 1.
(a) *length:5,* $\begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$ (b) *length:5,* $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (c) *length:5,* $(2/5, 3/5, 2\sqrt{3}/5)^T$

Exercise 2.
(a) $(4, \sqrt{3} + 2\sqrt{2} - 3, 4 - \sqrt{5})^T$
(b) $\begin{pmatrix} -2 \\ -19/4 \\ 12 \end{pmatrix}$

Exercise 3.
(a) $6 + (-6) = 0$
(b) $3 + (-4) + 1 = 0$

Exercise 4. $\Pi/4$

Exercise 5. $-13/21$

Exercise 6. $k = 7$

Exercise 7. $k = -4$

Exercise 8. $u \cdot u = ||u||^2$

Exercise 9. *v and w are perpendicular to each other.*

Exercise 10.
(a) $(1, -7, 11)^T$
(b) $(1, 3, -2)^T$
(c) $(0, 0, 0)^T$

Exercise 11.
$(1, 1, 2)^T \times (2, 0, 1)^T = (1, 3, -2)^T$
$(2, 0, 1)^T \times (1, 1, 2)^T = (-1, -3, 2)^T$

Exercise 12.
$(4, 1, 1)^T \times (-1, 0, 2)^T = (2, -9, 5)^T$.

Exercise 13.
$(-2, -4, -6)^T \times (3, -6, 1)^T = (-40, -16, 24)^T$.

Exercise 14.
There are three equations and two unknowns, i.e t and k.
$6 + 3t = 3$
$2 + t = 1$
$7 + 5t = k$
Solving this system of linear equations, $t = -1$ and $k = 2$ staisfy the three equations.

Exercise 15. *The vector* $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ *is normal to the line. Therefore the required vector is perpendicular to this vector. Swapping the components (works only for 2D) and negating only one of the components,* $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ *is a vector which is parallel to the line. Note that* $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 0$.

Exercise 16. *The implicit equation has the form* $ax + by - d = 0$. *Since the two lines are perpendicular to each other therefore their normals are perpendicular to each other too. Using the technique of the previous exercise,* $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$ *is a vector which is perpendicular to* $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$. *To find* d *we use the fact that* $(1, -1)$ *is on the line. Thus* $(-7)(1) + (3)(-1) - d = 0 \Longrightarrow d = -10$. *The line is:* $-7x + 3y = -10$.

Exercise 17. *If the two lines intersect each other, there should be a value for the parameter* s *and a value for the parameter* t *for which both* ℓ_1 *and* ℓ_2 *give the same point. Therefore there are three equations and two unknowns:*
$0 + s = -5 + 3t$
$-3 - s = -7 + 6t$
$6 - 5s = 2 + 2t$
But since there are no values for s *and* t *that satisfy all the equations, it means that the two lines do not intersect each other.*

Exercise 18. *Since the line passes through the origin,* p^T *is a vector parallel to the line. Therefore since* n *is a normal vector for* ℓ, $n \cdot p^T = 0$.

Exercise 19. $p^T - p'^T$ *is a vector that is parallel to the line. Therefore since* n *is a normal vector for* ℓ, $n \cdot (p^T - p'^T) = 0$. *Therefore* $n \cdot p^T - n \cdot p'^T = 0$.

Exercise 20. *We need two (linearly independent) vectors on the plane. We will choose* ${p_1}^T - {p_2}^T = (-8, 7, -2)^T$ *and* ${p_1}^T - {p_3}^T = (-12, -3, 6)^T$. *Note that we can simplify the second vector by dividing all the components by* -3. *Therefore:*
$(x, y, z)^T = (0, 7, 6)^T + s(-8, 7, -2)^T + t(4, 1, -2)^T$.

Exercise 21. *We use the cross product of the vectors computed in the previous exercise to find the normal vector.* $(4, 1, -2)^T \times (-8, 7, -2)^T = (12, 24, 36)$ *and we simplify it by dividing all the components by* 12. *Therefore* $(a, b, c)^T = (1, 2, 3)$. *To find* d *we substitue one of the points, for example* p_1. $(1)(0) + (2)(7) + (3)(6) = d \Longrightarrow d = 32$. *Thus the implicit equation of the plane is:*
$x + 2y + 3z = 32$.

Exercise 22.
$(1, 4, -1)^T \times (2, -3, 1)^T = (1, -3, -11)^T$
$(1)(-1) + (-3)(1) + (-11)(3) = d \Longrightarrow d = -37$
$x - 3y - 11z = -37$

Exercise 23.
$(x, y, z)^T = (1, 2, 3)^T + t(1, 1, 1)^T + s(2, 1, 1)^T$
$y - z = -1$

Exercise 24.
$z = 0$

Exercise 25.
$(2)(-1 + 2t) + (2 + 3t) - (3 + 4t) = 0 \Longrightarrow t = 1$
The intersection point is: $(1, 5, 7)$.

Exercise 26.
$(-3 - t) - 3(1 + 2t) + 2(2 + 3t) = 4 \Longrightarrow t = 0$
The intersection point is: $(-3, 1, 2)$.

Exercise 27.
The answer is yes. Find the implicit equations and verify easily that they describe the same plane.